Supplementary Materials

1 Case Studies Data

1.1 Example 1: Liquefied Natural Gas Plant

Table S1 Equipment dimensions for liquefied natural gas plant

Unit	Description	a(i)/m	b(i)/m	r(i)/m
1	PMR compressor LP suction drum	3.613	3.613	4.603
2	PMR compressor HP suction drum	3.217	3.217	4.9
3	PMR compressor	18.809	5.939	5.741
4	PMR compressor cooler	2.969	1.979	2.969
5	Overhead crane for PMR compressor	22.769	15.839	5.939
6	SW cooler 1	7.919	1.979	4.949
7	SW cooler 2	7.919	1.979	4.949
8	PMR receiver	4.157	4.157	9.8
9	LP precool exchanger	4.157	4.157	21.086
10	HP precool exchanger	4.355	4.355	21.779
11	Joule-Thomson valve 1	0.989	0.989	0.989
12	Joule-Thomson valve 2	0.989	0.989	0.989
13	MR separator 1	4.454	4.454	12.869
14	MCHE	5.642	5.642	41.579
15	MR compressor suction drum	5.444	5.444	8.909
16	MR compressor	17.126	5.939	5.939
17	MR compressor cooler	2.969	1.979	2.969
18	Overhead crane for MR Compressor	22.769	15.839	5.939
19	SW cooler 3	3.959	2.474	2.969
20	SW cooler 4	3.959	2.474	2.969
21	Joule-Thomson valve 3	1.484	1.484	1.484
22	Joule-Thomson valve 4	1.484	1.484	1.484

Table S2 Parameters of connection and pumping for example 1

Connection	C_{ij}^{c} (rmu/m)	C_{ij}^{h} (rmu/m)	C_{ij}^{v} (rmu/m)	OP_{ij} (m)	IP_{ij} (m)
2.3	150	750	7500	2.45	2.8705
3.6	150	750	7500	2.8705	2.4745
3.7	150	750	7500	2.8705	2.4745
7.2	150	750	7500	2.4745	2.45
8.1	150	750	7500	8.9	4
10.11	150	750	7500	18.8895	0.4945
11.1	150	750	7500	0.4945	18.8895
9.1	150	750	7500	4	18.8895
10.9	150	750	7500	18.8598	4
12.9	70	250	2500	0.4945	18.543
9.12	70	250	2500	18.453	0.4945
13.14	150	750	7500	4	12
14.15	150	750	7500	4	4
15.16	150	750	7500	8.4545	2.9695
16.17	150	750	7500	2.9795	1.4845
17.16	150	750	7500	1.4845	2.9695
14.21	70	250	2500	28	0.742
21.14	70	250	2500	0.742	28
16.19	150	750	7500	2.9695	1.4845
19.16	150	750	7500	1.4845	2.9695
16.2	150	750	7500	2.9695	1.4845
20.16	150	750	7500	1.4845	2.9695
22.14	150	750	7500	0.742	36.7895
14.22	150	750	7500	36.7895	0.742
10.2	150	750	7500	12	2.45
9.1	150	750	7500	12	2.3015
6.8	150	750	7500	2.4745	8.9
14.9	150	750	7500	20	12
10.13	150	750	7500	4	4

Table S3 Other parameters for example 1

parameter	value
K	8
FC1（rmu)	4600
FC2 (rmu/sqm)	33.3
LC (rmu/sqm)	666
FH (m)	8

1.2 Example 2: Crude oil and gas processing plant

Table S4 Equipment dimensions for crude oil and gas processing plant

Unit	Description	a(i)/m	b(i)/m	r(i)/m
1	Separator	20	5	5
2	KO drum	16	4	4
3	Contactor 1	3	3	8
4	Contactor 2	2	2	5.333
5	Compressor 1	8	3	3
6	Compressor 2	5.333	2	2
7	Mixer 1	1	1	1
8	Heat exchanger 1	8	2.5	2.5
9	Heat exchanger 2	9.6	3.6	3.6
10	Heat exchanger 3	14.4	4.5	4.5
11	Contactor 3	3	3	9
12	Contactor 4	3.6	3.6	10.8
13	Contactor 5	5.4	5.4	16.2
14	Compressor 3	6.4	2.4	2.4
15	Compressor 4	7.68	2.88	2.88
16	Compressor 5	11.52	4.32	4.32
17	Heat exchanger 4	6	1.875	1.875
18	Heat exchanger 5	7.2	2.7	2.7
19	Heat exchanger 6	10.8	3.375	3.375
20	Contactor 6	2.4	2.4	7.2
21	Contactor 7	2.88	2.88	8.64
22	Contactor 8	4.32	4.32	12.96
23	Compressor 6	5.76	2.16	2.16
24	Compressor 7	6.912	2.592	2.592
25	Compressor 8	10.368	3.888	3.888

Table S5 Parameters of connection and pumping for example 2

Connection	C_{ij}^{c} (rmu/m)	C_{ij}^{h} (rmu/m)	C_{ij}^{v} (rmu/m)	OP_{ij} (m)	IP_{ij} (m)
1.7	50	225	2250	0	1
1.2	200	900	9000	0	2
2.4	72	324	3240	0	2.667
4.6	72	324	3240	5.333	2
6.7	72	324	3240	2	1
5.7	108	486	4860	3	1
2.3	108	486	4860	0	4
3.5	108	486	4860	8	3
7.8	57.5	258.75	2587.5	0	2.5
7.9	69	310.5	3105	0	3.6
7.1	103.5	465.75	4657.5	0	4.5
8.11	57.5	258.75	2587.5	0	4.5
11.14	46	207	2070	9	2.4
14.17	46	207	2070	2.4	1.875
17.2	46	207	2070	0	3.6
20.23	41.4	186.3	1863	7.2	2.16
9.12	69	310.5	3105	0	5.4
12.15	55.2	248.4	2484	10.8	2.88
15.18	55.2	248.4	2484	2.88	2.7
18.21	55.2	248.4	2484	0	4.32
21.24	49.68	223.56	2235.6	8.64	2.592
10.13	103.5	465.75	4657.5	0	8.1
13.16	82.8	372.6	3726	16.2	4.32
16.19	82.8	372.6	3726	4.32	3.375
19.22	82.8	372.6	3726	0	6.48
22.25	74.52	335.34	3353.4	12.96	3.888

Table S6 Other parameters for example 2

parameter	value
K	8
FC1（rmu)	3330
FC2 (rmu/sqm)	33.3
LC (rmu/sqm)	666
FH (m)	5

1.3 Example 3: Du62

Table S7 Equipment dimensions for example 3

Unit	a(i)/m	b(i)/m	r(i)/m
1	14.49	14.49	19
2	11.40	11.40	12
3	14.97	14.97	20
4	16.12	16.12	13
5	14.42	14.42	1
6	17.15	17.15	3
7	17.97	17.97	6
8	16.31	16.31	17
9	21.00	21.00	15
10	18.44	18.44	6
11	11.96	11.96	10
12	12.96	12.96	7
13	18.49	18.49	10
14	18.89	18.89	18
15	20.49	20.49	18
16	12.12	12.12	15
17	19.49	19.49	12
18	12.00	12.00	6
19	13.67	13.67	10
20	15.49	15.49	7
21	11.83	11.83	8
22	17.44	17.44	3
23	17.32	17.32	5
24	12.73	12.73	15
25	15.87	15.87	3
26	14.00	14.00	14
27	13.27	13.27	6
28	12.00	12.00	18
29	14.87	14.87	20
30	11.40	11.40	1
31	13.49	13.49	4
32	11.66	11.66	20
33	19.97	19.97	4
34	14.49	14.49	14
35	12.25	12.25	5
36	10.39	10.39	9

37	18.89	18.89	13
38	12.00	12.00	3
39	15.87	15.87	19
40	11.62	11.62	4
41	14.28	14.28	12
42	14.28	14.28	9
43	9.95	9.95	13
44	12.65	12.65	4
45	18.89	18.89	3
46	16.12	16.12	5
47	13.78	13.78	15
48	16.73	16.73	10
49	13.42	13.42	4
50	10.20	10.20	5
51	14.07	14.07	11
52	12.65	12.65	17
53	14.14	14.14	13
54	19.00	19.00	18
55	15.20	15.20	16
56	11.83	11.83	20
57	8.77	8.77	11
58	13.67	13.67	6
59	15.20	15.20	12
60	9.54	9.54	6
61	14.49	14.49	7
62	16.49	16.49	17

The Table S8 contains parameters of connection, pumping, the points of input and output, which can be found in the excel file of the Supplementary Materials.

Table S9 Other parameters for example 3

parameter	value
K	4
FC1（rmu)	3330
FC2 (rmu/sqm)	33.3
LC (rmu/sqm)	666
FH (m)	5

1.4 Example 4: LNG plant

Table S10 Equipment dimensions for LNG plant

Unit	Description	a(i)/m	b(i)/m	r(i)/m
1	Inlet scrubbing tower	2	2	10
2	Inlet filter	0.8	0.8	2.6
3	Intake heat exchanger	1.5	1.5	12
4	Absorber	3.6	3.6	24
5	Flash drum	2.4	2.4	6
6	Sweet gas separator	2	2	10
7	Amine-rich pre-filter	0.4	0.4	2.2
8	Activated carbon filter	1.6	1.6	6
9	Amine-rich trim filter	0.4	0.4	2.2
10	High Pressure Amine Pump	1.3	0.5	1
11	Regenerative gas scrubber	1.4	1.4	6
12	Molecular Sieve Dryer	3.2	3.2	9.5
13	Molecular Sieve Dryer	3.2	3.2	9.5
14	Molecular Sieve Dryer	3.2	3.2	9.5
15	Molecular Sieve Dryer	3.2	3.2	9.5
16	Mercury removal tower	3	3	7
17	Mercury removal tower	3	3	7
18	Dry gas filter	1.3	1.3	4.6
19	Dust filter	1.3	1.3	4.6
20	Lean/rich amine heat exchanger	0.9	0.9	6
21	Lean/rich amine heat exchanger	0.9	0.9	6
22	A regenerator	2.2	2.2	24
23	Tower top condenser	0.7	0.7	5.5
24	Tower top reflux drum	1	1	3
25	Reflux pump	1.3	0.5	1
26	Reboiler	1.7	1.7	6
27	Low Pressure Amine Pump	1.3	0.5	1
28	Lean amine cooler	0.7	0.7	5.5
29	Virtual equipment 1	1.5	1.5	0.1
30	Virtual equipment 2	1.5	1.5	0.1
31	Virtual equipment 3	1.5	1.5	0.1
32	Virtual equipment 4	1.5	1.5	0.1
33	Virtual equipment 5	1.5	1.5	0.1

Table S11 Parameters of connection and pumping for example 4

Connection	C_{ij}^{c} (rmu/m)	C_{ij}^{h} (rmu/m)	C_{ij}^{v} (rmu/m)	OP_{ij} (m)	IP_{ij} (m)
1.2	250	1000	10000	5	1.3
2.3	250	1000	10000	1.3	6
3.4	250	1000	10000	6	12
3.6	250	1000	10000	6	5
4.3	250	1000	10000	12	6
4.5	250	1000	10000	12	3
5.7	250	1000	10000	3	1.1
5.2	250	1000	10000	3	3
6.5	250	1000	10000	5	3
6.11	250	1000	10000	5	3
7.8	250	1000	10000	1.1	3
8.9	250	1000	10000	3	1.1
9.2	250	1000	10000	1.1	3
10.4	250	1000	10000	0.5	12
11.12	250	1000	10000	3	4.75
11.13	250	1000	10000	3	4.75
11.14	250	1000	10000	3	4.75
11.15	250	1000	10000	3	4.75
12.18	250	1000	10000	4.75	2.3
13.18	250	1000	10000	4.75	2.3
14.18	250	1000	10000	4.75	2.3
15.18	250	1000	10000	4.75	2.3
16.19	250	1000	10000	3.5	2.3
17.19	250	1000	10000	3.5	2.3
18.16	250	1000	10000	2.3	3.5
18.17	250	1000	10000	2.3	3.5
20.21	250	1000	10000	3	3
20.22	250	1000	10000	3	12
20.27	250	1000	10000	3	0.5
21.2	250	1000	10000	3	3
21.22	250	1000	10000	3	12
22.21	250	1000	10000	12	3
22.23	250	1000	10000	12	2.75
22.26	250	1000	10000	12	3
23.24	250	1000	10000	2.75	1.5
24.25	250	1000	10000	1.5	0.5
25.22	250	1000	10000	0.5	12
26.22	250	1000	10000	3	12
27.28	250	1000	10000	0.5	2.75

Table S12 Other parameters for example 4

parameter	value
K	8
FC1（rmu)	3330
FC2 (rmu/sqm)	33.3
LC (rmu/sqm)	666
FH (m)	5
x'(i), y'(i)	(5,5)

Table S13 Set of equipment for example 4

set	equipment number
tower	22
cond	23,24,25
rebo	26

2 Layouts

2.1 Example 2

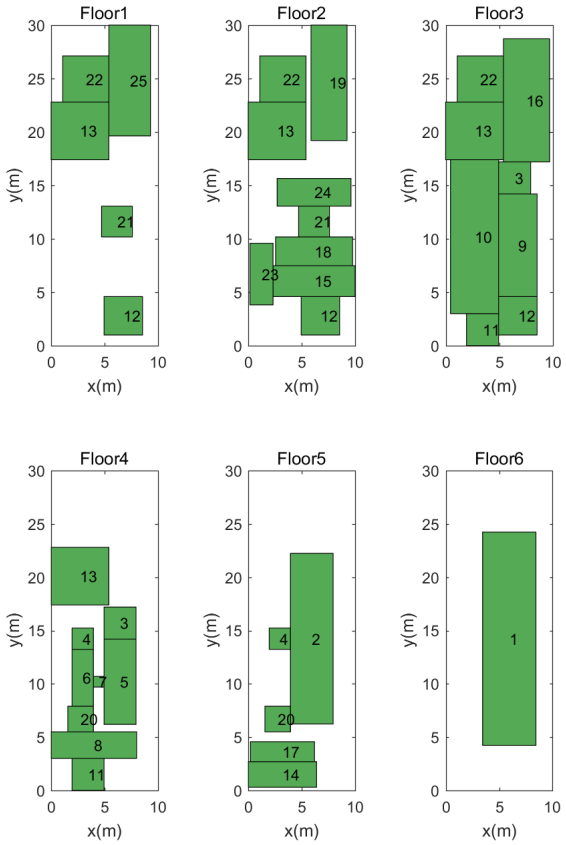

Figure S1 Optimal layout for example 2: Simultaneous Approach

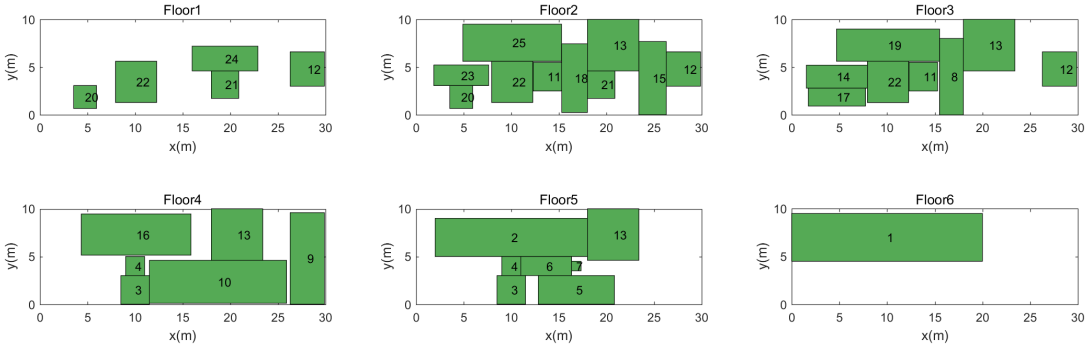

Figure S2 Optimal layout for example 2: Multi-directional Search Approach

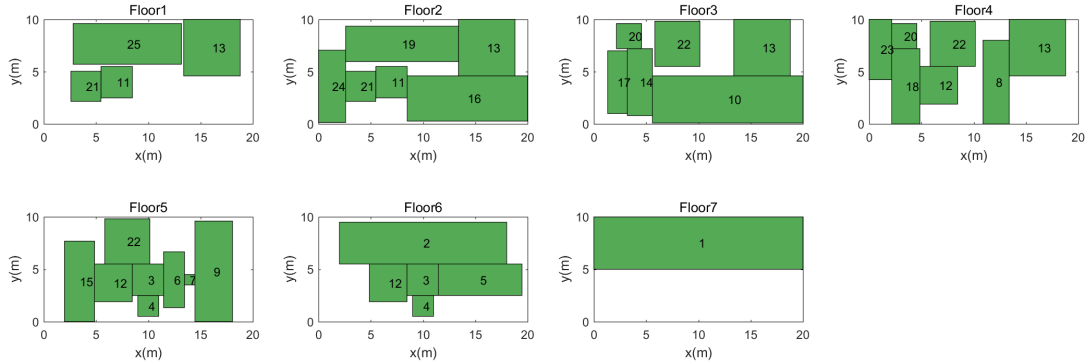

Figure S3 Optimal layout for example 2: Iterative Downsizing Approach

2.2 Example 4

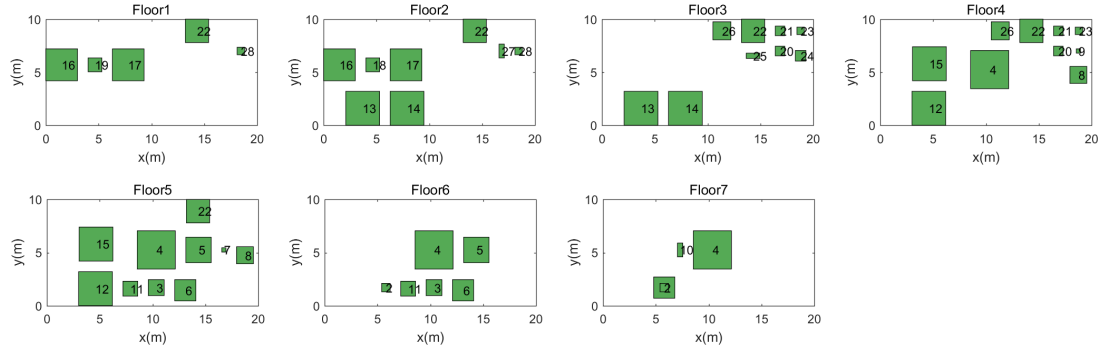

Figure S4 Optimal layout for example 4: Simultaneous Approach

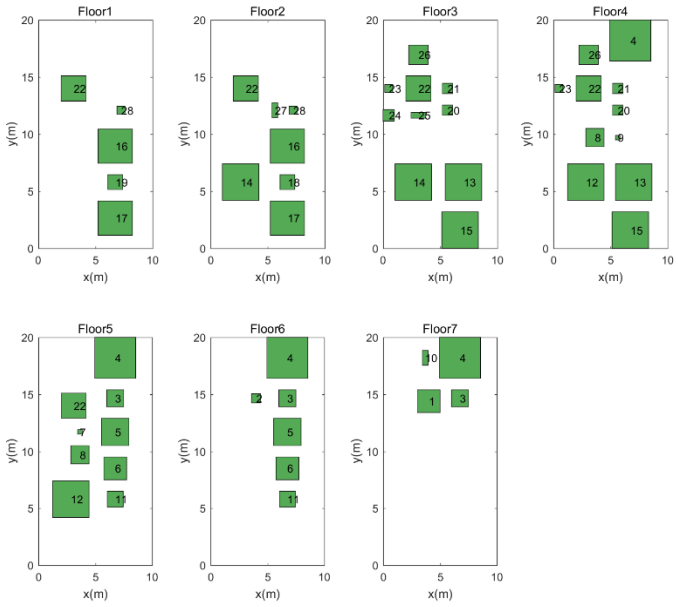

Figure S5 Optimal layout for example 4: Multi-directional Search Approach

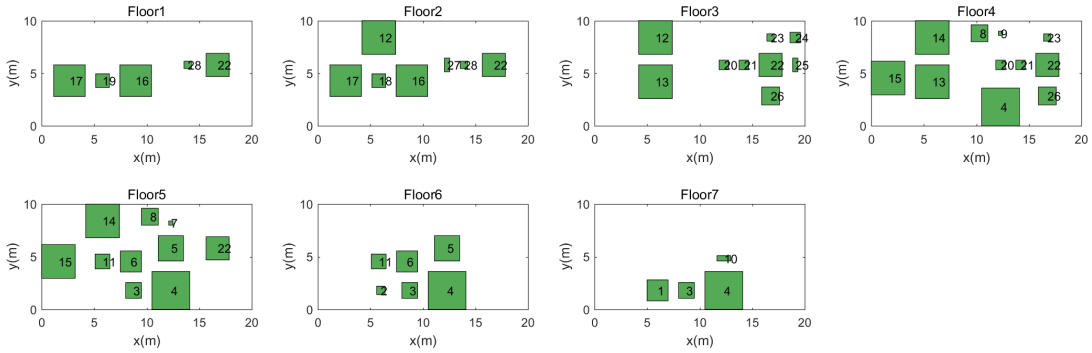

Figure S6 Optimal layout for example 4: Iterative Downsizing Approach

Nomenclature

Indices

i, j	equipment item
k	floor number
s	candidate floor specification
n	model version number

Sets

I	set of equipment items
I’	set of virtual equipment items
MF	set of multi-floor equipment items
RF	set of floors with reserved space
pump	set of pump items
tower	set of towers
cond	set of condensers
rebo	set of reboilers

Parameters

α_i, β_i, γ_i	dimensions of item i
FH	floor height
f_{ij}	1, if material flows from equipment item i to j; 0, otherwise
C_{ij}^{c}	connection cost between items i and j
C_{ij}^{v}	vertical pumping cost of connection i-j

C_{ij}^{h}	horizontal pumping cost of connection i-j
FC1	fixed floor construction cost
FC2	area-dependent floor cost
LC	land cost
$D_{ij}^{\min}$	minimum distance between equipment items i and j
OP_{ij}	distance between the base and output point on equipment i for connection i-j
IP_{ij}	distance between the base and input point on equipment j for connection i-j
M_i	number of floors required by item i
$\overline{X_s}$, $\overline{Y_s}$	x-y dimensions of candidate floor specification
x_i', y_i'	coordinates of the geometrical center of virtual item i
AR_s	area of candidate floor specification
GP	the difference in the number of auxiliary data between the subproblem A_n and A_{n-1}.
$STEP$	the difference in the number of auxiliary data between the subproblem A_n and A_{n-1}.
S	The number of the elements s in the candidate floor specification set

Integer Variables

NF	number of floors required for layout

Binary Variables

V_{ik}	1, if item i is assigned to floor k

N_{ij}	1, if items i and j occupy a same floor; 0, otherwise
n_{ijk}	1, if items i and j are assigned to floor k; 0, otherwise
S_{ik}^{s}	1, if item i begins at floor k; 0, otherwise
S_{ik}^{f}	1, if item i terminates at floor k; 0, otherwise
W_k	1, if floor k is occupied; 0, otherwise
O_i	1, if length of item i is equal to α_i; 0, otherwise
$E1_{ij}, E2_{ij}$	non-overlapping binary variables, which prevent equipment from overlapping in the x-y plane
Q_s	1, if candidate floor specification s is selected for the layout; 0, otherwise

Continuous Variables

l_i	length of item i
d_i	breadth of item i
h_i	height of item i
x_i, y_i, z_i	coordinates of the geometrical center of item i
R_{ij}	relative distance in x coordinates between items i and j, if i is to the right of j
L_{ij}	relative distance in x coordinates between items i and j, if i is to the left of j
A_{ij}	relative distance in y coordinates between items i and j, if i is above j
B_{ij}	relative distance in y coordinates between items i and j, if i is below j
U_{ij}	relative distance in z coordinates between items i and j, if i is higher

	than j
D_{ij}	relative distance in z coordinates between items i and j, if i is lower than j
TD_{ij}	total rectilinear distance between items i and j
FA	floor area
$X^{\max}$， $Y^{\max}$	dimensions of the selected floor
NQ_s	linearization variable expressing the product of Q_s and NF